Transformation of Functions

Key Points:

- Even functions are symmetric about the *y*-axis, whereas odd functions are symmetric about the origin.
- Even functions satisfy the condition $f(x) = f(-x)$
- Odd functions satisfy the condition $f(x) = -f(-x)$
- A function can be odd, even, or neither.
- The summary of transformations written via equations is as follows:

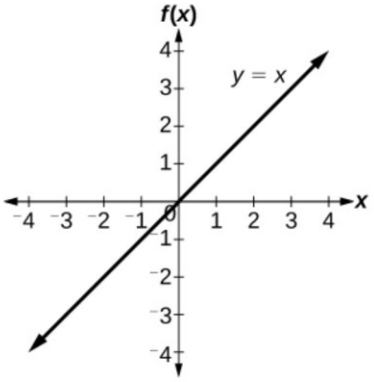
Vertical shift	$g(x) = f(x) + k$ (up for $k > 0$)
Horizontal shift	$g(x) = f(x - h)$ (right for $h > 0$)
Vertical reflection	$g(x) = -f(x)$
Horizontal reflection	$g(x) = f(-x)$
Vertical stretch	$g(x) = af(x)$ ($a > 1$)
Vertical compression	$g(x) = af(x)$ $(0 < a < 1)$
Horizontal stretch	$g(x) = f(bx)$ $(0 < b < 1)$
Horizontal compression.	$g(x) = f(bx)$ ($b > 1$)

When combining horizontal transformations written in the form $f(bx - h)$, first horizontally shift by $\frac{h}{b}$ and then horizontally stretch by $\frac{1}{b}$.

When combining horizontal transformations written in the form $f(b(x - h))$, first horizontally stretch by $\frac{1}{b}$ and then horizontally shift by h.

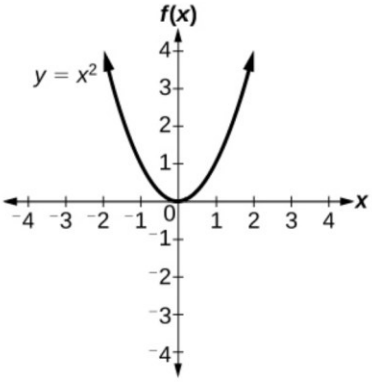
Tool Kit Functions:

Identity



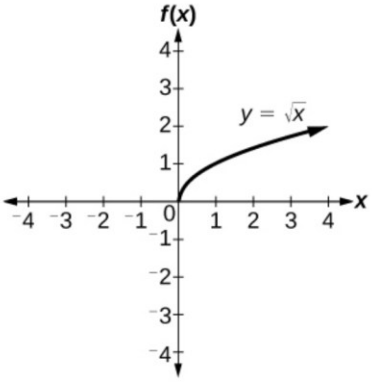
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Square



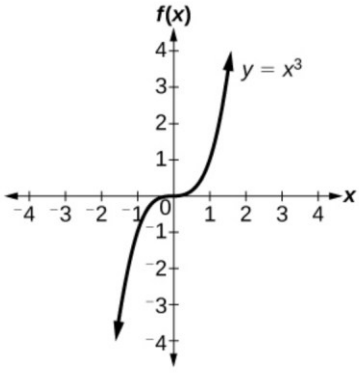
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Square Root



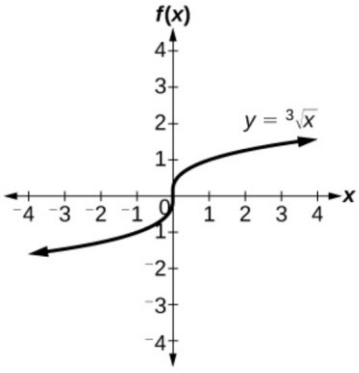
Domain: $[0, \infty)$
Range: $[0, \infty)$

Cubic



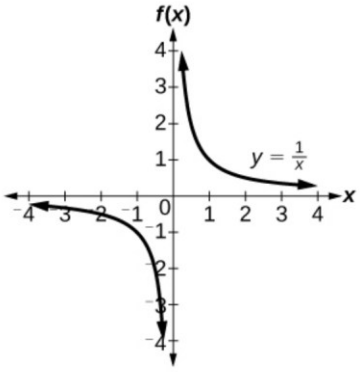
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Cube Root



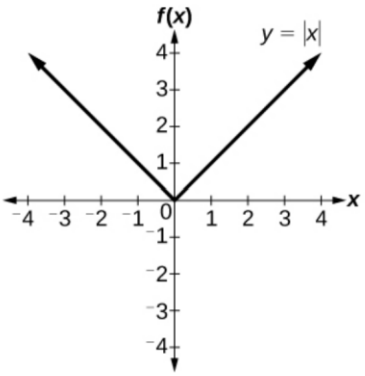
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Reciprocal



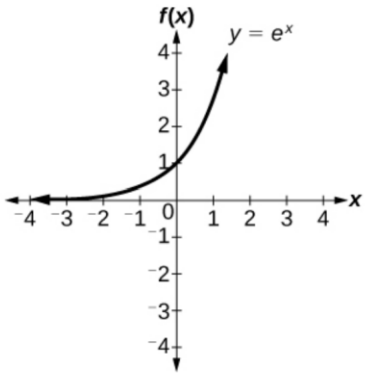
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

Absolute Value



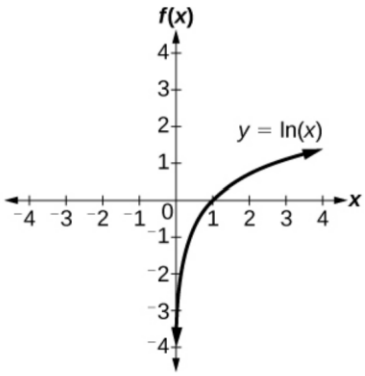
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Exponential



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Natural Logarithm



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

Transformation of Functions Videos

- Identifying Horizontal and Vertical Shifts: Examples 1-3
- Reflecting a Graph Horizontally and Vertically: Examples 4-6
- Determining whether a function is Even, Odd or Neither: Example 7
- Identifying Vertical Stretches and Compressions: Examples 8-9
- Identifying Horizontal Stretches and Compressions: Example 10
- Combining Transformations: Examples 11-12

Practice Exercises

Follow the directions for each exercise below:

1. Sketch a graph of the given function: $f(x) = (x-3)^2$

2. Sketch a graph of the given function: $f(x) = (x+4)^3$

3. Sketch a graph of the given function: $f(x) = \sqrt{x} + 5$

4. Sketch a graph of the given function: $f(x) = -x^3$

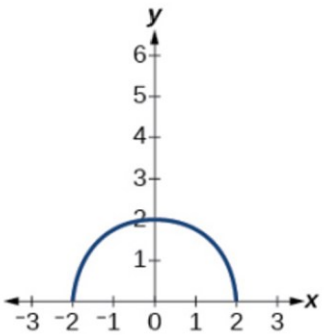
5. Sketch a graph of the given function: $f(x) = \sqrt[3]{-x}$

6. Sketch a graph of the given function: $f(x) = 5\sqrt{-x} - 4$

7. Sketch a graph of the given function: $f(x) = 4[|x-2| - 6]$

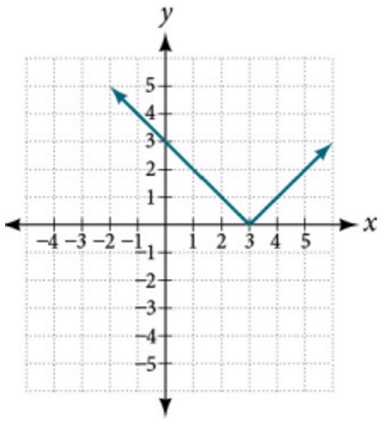
8. Sketch a graph of the given function: $f(x) = -(x+2)^2 - 1$

For #9-10, Sketch the graph of the function *g* if the graph of the function *f* is shown in Figure.

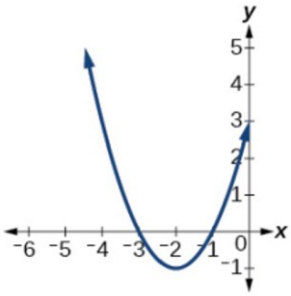


9. $g(x) = f(x-1)$
10. $g(x) = 3f(x)$

11. Write the equation for the standard function represented by the graph below:



12. Write the equation for the standard function represented by the graph below:

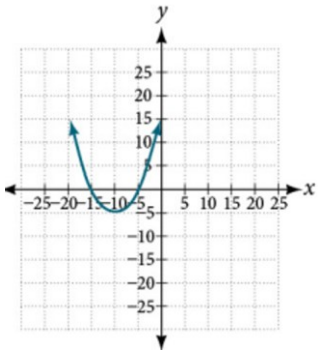


13. Determine whether the given function is even, odd, or neither: $f(x) = 3x^4$

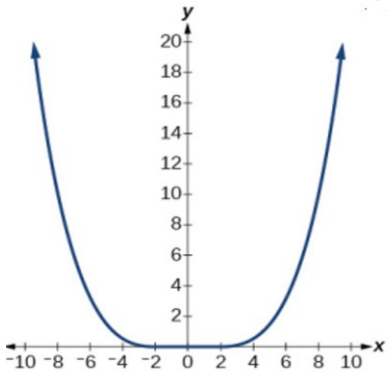
14. Determine whether the given function is even, odd, or neither: $f(x) = \sqrt{x}$

15. Determine whether the given function is even, odd, or neither: $f(x) = \frac{1}{x} + 3x$

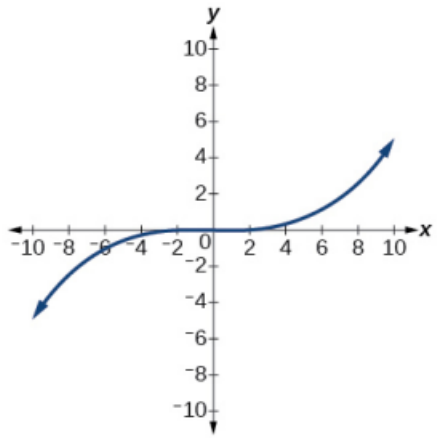
16. Analyze the graph below and determine whether the graphed function is even, odd, or neither:



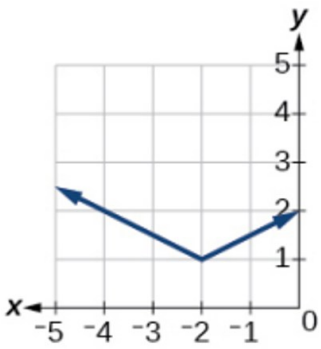
17. Analyze the graph below and determine whether the graphed function is even, odd, or neither:



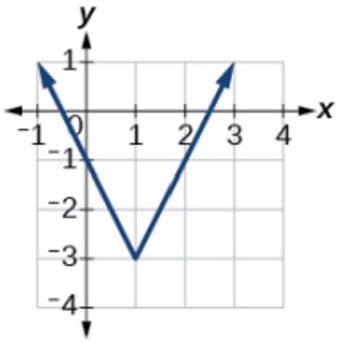
18. Analyze the graph below and determine whether the graphed function is even, odd, or neither:



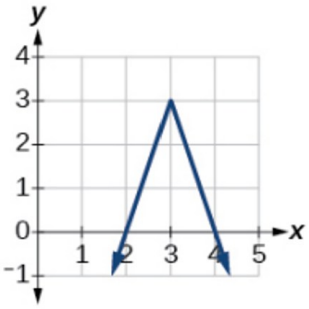
19. Write an equation for the transformation of $f(x) = |x|$:



20. Write an equation for the transformation of $f(x) = |x|$:

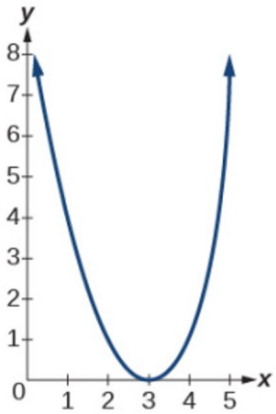


21. Write an equation for the transformation of $f(x) = |x|$:

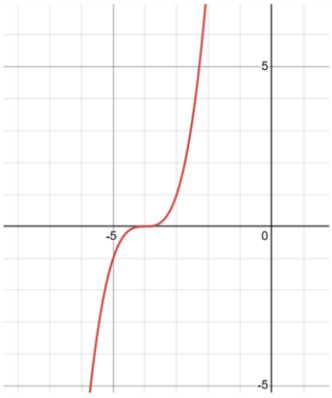


Answers:

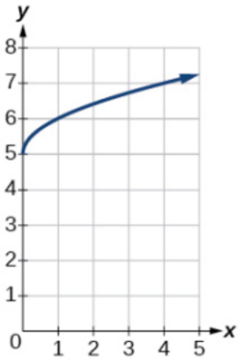
1.



2.



3.



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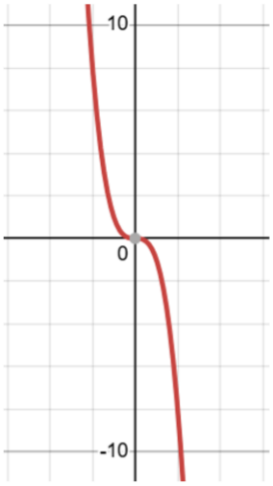

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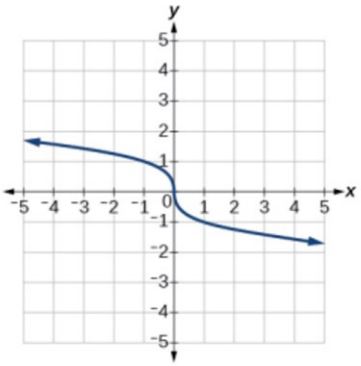

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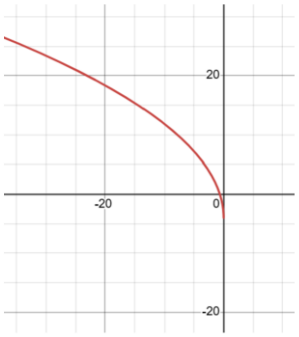

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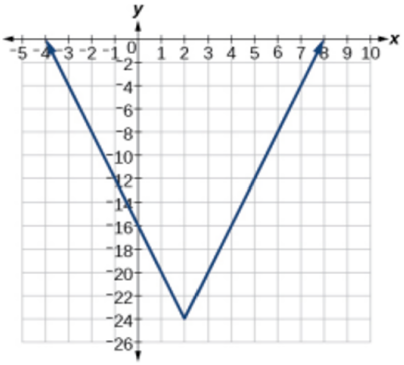
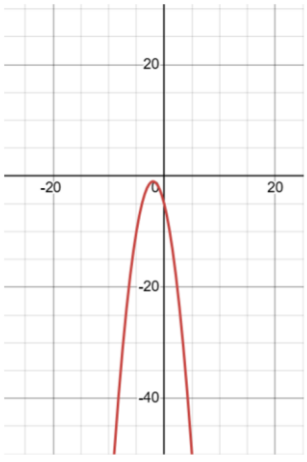
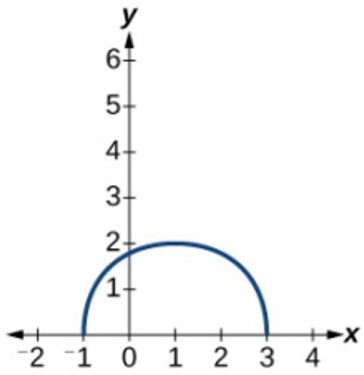


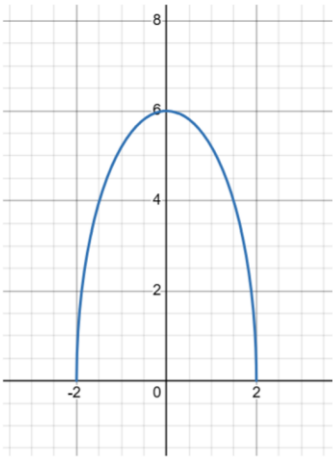
8.



9.



10.



11. $f(x) = |x - 3|$

12. $f(x) = (x + 2)^2 - 1$

13. Even

14. Neither

15. Odd

16. Neither

17. Even

18. Odd

19. $f(x) = \frac{1}{2}|x + 2| + 1$

20. $f(x) = 2|x - 1| - 3$

21. $f(x) = -3|x - 3| + 3$